approximation by Integerals

Trapezoidal rule:-

Rather than approximating the area bounded by a function by rectangle, one may use other shapes. for example, one may use trapezoids. The area of the trapezoid with corners at

$(a, 0), (b, 0), (a, f(a))$, and $(b, f(b))$ is

$\frac{1}{2}(f(a) + f(b))(b-a)$.

The trapezoidal approximation to $\int_a^b f(x)\,dx$. with

N Subdivisions is

$$T = \sum_{i=0}^{N-1} \frac{1}{2}\left(f(x_i) + f(x_{i+1})\right)\Delta$$

where $\Delta = \frac{b-a}{N}$ and $x_i = a + i\Delta$

ex:- Compute the trapezoidal approximation to

$\int_1^9 x^2\,dx$ with 4 Subdivisions.

Sol^n:-

$$T = \sum_{i=0}^{3} \tfrac{1}{2}\left((1+i2)^2 + (1+(i+1)2)^2\right)2$$

$$= \frac{1}{2}(1^2 + 3^2 + 3^2 + 5^2 + 5^2 + 7^2 + 7^2 + 9^2)2$$

$$= \tfrac{1}{2}(1 + 9 + 9 + 25 + 25 + 49 + 49 + 81)$$

$$= 248$$

Simpson's rule :-

Simpson's rule for approximating integrals is ~~based~~ based on approximating $\int_{x_i}^{x_{i+1}} f(x)\,dx$ by the ~~area~~ area bounded by the Parabola passing through $(x_i; f(x_i))$, $\left(\frac{x_i+x_{i+1}}{2}, f\left(\frac{x_i+x_{i+1}}{2}\right)\right)$ and $(x_{i+1}, f(x_{i+1}))$

Rather than deriving Simpson's rule from its geometric description, we write the Simpson's rule approximation in terms of the midpoint and trapezoidal approximations.

$$\boxed{S = \frac{2}{3}M + \frac{1}{3}T}$$

Another formula for Simpson's rule.

Directly.

$$S = \frac{1}{6}\left[f(a) + 4f(a+\tfrac{1}{2}\Delta) + 2f(a+\Delta) + 4f(a+\tfrac{3}{2}\Delta) + \right.$$
$$\left. \ldots + 4f(a+[N-\tfrac{1}{2}]\Delta) + f(b)\right]\Delta$$

$$= \frac{1}{6}\left[f(a) + 4\sum_{i=0}^{N-1} f(a+(i+\tfrac{1}{2})\Delta)\right) +$$
$$2\left(\sum_{i=1}^{N-1} f(a+i\Delta)\right) + f(b)\Big]\Delta.$$

ex:- Compute the Simpson's rule approximation to $\int_1^9 x^2 dx$ with N=4. Subdivisions.

Sol^n:- $S = \cdot 1/6[1^2+4(2^2+4^2+6^2+8^2)+ 2(3^2+5^2+7^2)+9^2]2$

$= 1/3[1+4(4+16+36+64)+2(9+25+49)+81]$

$= 1/3[1+480+166+81]$

$= 1/3[656]$

$= 242\frac{2}{3}$